

Manifestation of proton structure in the initial-state anisotropies in high-energy proton-proton collisions

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Abstract

Ridge-like correlations in high-energy proton-proton collisions reported by the CMS collaboration suggest a collective flow that resembles the one in heavy-ion collisions. If the hydrodynamic description is valid then the effect results from the initial anisotropy of the colliding matter which depends on the structure of protons. Following recent theoretical developments, we propose several phenomenological models of the proton structure and calculate the anisotropy coefficients using the Monte Carlo Glauber model. Our estimates suggest that the event multiplicity dependence allows one to discriminate between different proton models.

Keywords: high-energy proton-proton collisions, two-particle correlations, collective flow, proton structure, renormalization group

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1 Introduction

The analysis of two-particle angular correlations in pp collisions at $\sqrt{s} = 7$ TeV revealed an unexpected near side ($\Delta\phi = 0$) correlation in the azimuthal angle of produced particles ('ridge effect') [1]. There is no obvious reason why such a long-range in pseudorapidity correlation should occur [2, 3]. Ridge effect has been previously observed in relativistic heavy-ion collisions and it was explained by a collective anisotropic flow of hot and dense medium (hadronic gas or quark gluon plasma) created at the collision.

It is possible that the hydrodynamic explanation applies to high-multiplicity pp events which are sufficiently energetic for the collision products to thermalize [4]. In this paper we assume validity of such a scenario and postulate that the proton internal structure can manifest itself in the ridge effect. During the hydrodynamic expansion, the spatial anisotropies of the collision area are transformed into anisotropic flow of produced particles. Thus, the spatial anisotropies due to the proton structure should be reflected in the measurable collective flow coefficients.

Actually, there exist in literature estimates of the possible elliptic flow in pp collisions [5–13]. We basically follow the approach of [5] in order to make a link between the proton models considered by us and the experimental data. Within the framework of Glauber model [14], we calculate for each model the initial spatial anisotropies: eccentricity, denoted by ϵ_2 , and triangularity, denoted by ϵ_3 , in proton-proton collision taking into account event-by-event fluctuations in the proton density profile.

The considered phenomenological proton densities are inspired by the quark-diquark [17] and RGPEP [18] models of proton. We find that it is possible to distinguish highly eccentric (rod-like) and highly triangular (triangle-like) fixed proton densities by looking at distributions of ϵ_2 and ϵ_3 and the multiplicity dependence of their RMS values. We also calculate these quantities for the fluctuating anisotropic proton configurations (Gaussian-fluctuating model).

We do not explicitly present results for the anisotropic flow coefficients v_n because we limit our discussion to comparison of predictions of different models for the initial states in hydrodynamic evolution and the prediction of the exact shapes of the resulting final flow anisotropies is beyond the scope of this work. Our calculations concern the initial spatial anisotropies, though we emphasize that they should correspond to the flow coefficients measurable in multi-particle correlations in a fashion that is unlikely to be far from linear [15, 16].

2 Glauber model for pp collisions

The optical limit of Glauber model treats the collision of two composite particles as a superposition of independent binary collisions between their constituents. In case of

pp collisions, we assume the constituents are partons of one type, and their smoothed distribution is given by proton density profile. We fix the partonic cross section $\sigma_{gg} = 4.3 \text{ mb}$ [15] and adjust the number of partons in proton N_g so that the experimental inelastic pp cross section $\sigma_{pp} \approx 60 \text{ mb}$ [19] is reproduced.

The geometrical quantities calculated within the Glauber formalism correspond to the impact plane of the collision (xy) such that any z -dependence is integrated out. The density of binary collisions is given by the expression:

$$n_{\text{coll}}(x, y; b, \Sigma_A, \Sigma_B) = \sigma_{gg} \int_{-\infty}^{\infty} dz \rho \left(x - \frac{b}{2}, y, z; \Sigma_A \right) \int_{-\infty}^{\infty} dz' \rho \left(x + \frac{b}{2}, y, z'; \Sigma_B \right), \quad (1)$$

where $\rho(x, y, z; \Sigma)$ is a proton density profile, b is an impact parameter and by Σ we mean a set of fluctuating parameters describing the proton internal structure and orientation in space (for fixed configuration models Σ dependence will correspond only to possible rotations in space) [14].

For each event characterized by b , Σ_A and Σ_B we calculate eccentricity ϵ_2 and triangularity ϵ_3 . We choose the following definition of spatial anisotropy [20]:

$$\epsilon_n = \frac{\sqrt{\{s^n \cos(n\phi)\}^2 + \{s^n \sin(n\phi)\}^2}}{\{s^n\}}, \quad (2)$$

where ϕ is an azimuthal angle in xy plane and $s^2 = x^2 + y^2$. The curly brackets $\{\dots\}$ stand for the average taken with respect to the impact plane binary collision density $n_{\text{coll}}(x, y; b, \Sigma_A, \Sigma_B)$:

$$\{f(x, y)\} = \frac{\int dx dy f(x, y) n_{\text{coll}}(x, y; b, \Sigma_A, \Sigma_B)}{\int dx dy n_{\text{coll}}(x, y; b, \Sigma_A, \Sigma_B)} \quad (3)$$

The differential cross section (probability) of a given event depends only on the integrated number of binary collisions $N_{\text{coll}}(b, \Sigma_A, \Sigma_B) = \int dx dy n_{\text{coll}}(x, y; b, \Sigma_A, \Sigma_B)$,

$$\sigma(b, \Sigma_A, \Sigma_B) = 1 - \left[1 - \frac{N_{\text{coll}}(b, \Sigma_A, \Sigma_B)}{N_g^2} \right]^{N_g^2}. \quad (4)$$

The expectation value of a quantity Q is calculated in the following way

$$\langle Q \rangle = \frac{1}{\sigma_{pp}} \int_0^\infty 2\pi b db \int P(\Sigma_A) d\Sigma_A \int P(\Sigma_B) d\Sigma_B \sigma(b, \Sigma_A, \Sigma_B) Q(b, \Sigma_A, \Sigma_B), \quad (5)$$

where $\sigma_{pp} = \int_0^\infty 2\pi b db \int P(\Sigma_A) d\Sigma_A \int P(\Sigma_B) d\Sigma_B \sigma(b, \Sigma_A, \Sigma_B)$ and $P(\Sigma)$ is the probability density of proton configuration Σ .

We calculate charged hadron multiplicity N for each event assuming its linear scaling

with N_{coll} :

$$N = \alpha N_{\text{coll}}, \quad (6)$$

where α is determined by the demand of reproducing experimental minimum bias charged hadron multiplicity $\langle N \rangle = 30$ [21].

There is another approach to estimating the multiplicities and geometrical quantities within the Glauber formalism, namely the wounded nucleon model [22]. Within this model, all the averages denoted by $\{\dots\}$ are taken with respect to the local density of participating constituents instead of the density of binary collisions. However, in high energy hadronic collisions, multiple gluonic interactions are expected to occur, so we think it is more natural to use here the density of binary partonic collisions.

3 Models of proton internal structure

The renormalization group procedure for effective particles (RGPEP) [18] suggests that a proton can be described in terms of three effective quarks and a gluon body interacting via harmonic potential. For every considered special case of this picture we model the proton as a sum of isotropic 3D Gaussian densities ρ_q corresponding to effective quarks and anisotropic Gaussians ρ_g representing the gluonic flux tubes connecting the quarks:

$$\rho_q(\mathbf{r}; r_q) = \frac{1}{(2\pi)^{3/2} r_q^3} e^{-\frac{r^2}{2r_q^2}}, \quad (7)$$

$$\rho_g(\mathbf{r}; r_s, r_l) = \frac{1}{(2\pi)^{3/2} r_s^2 r_l} e^{-\frac{x^2+y^2}{2r_s^2} - \frac{z^2}{2r_l^2}}. \quad (8)$$

The effective quarks and the gluon body are clusters of partons, as in the two stage and cluster models [23–25] used to explain the shape of deep inelastic scattering structure functions.

3.1 Fixed configuration models: I and Y

According to [17], the ridge effect could be the consequence of large eccentricities ϵ_2 in high-multiplicity collisions between aligned quark-diquark states of protons which constitute around 20% of all possible states.

We model this state, labelling it with the symbol I, as a superposition of two effective quark bodies, with quark and diquark parton densities generically described in terms of simple functions, and two gluonic tubes connecting the quark bodies (Fig. 1). Such greatly simplified density model is sufficient for tracing the effects of symmetric rod-like shape of proton in comparison with other conceivable shapes (see below), leaving the more subtle effects of the asymmetry between quark and diquark for further analysis

elsewhere. Thus, we use here

$$\begin{aligned} \rho_I(\mathbf{r}) = & N_g \frac{1-\kappa}{2} \left[\rho_q \left(x, y, z - \frac{d}{2}; r_q \right) + \rho_q \left(x, y, z + \frac{d}{2}; r_q \right) \right] \\ & + N_g \frac{\kappa}{2} \left[\rho_g \left(x, y, z - \frac{d}{4}; r_q, \frac{d}{4} \right) + \rho_g \left(x, y, z + \frac{d}{4}; r_q, \frac{d}{4} \right) \right]. \end{aligned} \quad (9)$$

The free parameters of the model are: the effective quark radius r_q , the length of quark-diquark tube d , and the percentage of gluon body content κ . Assuming that only effective quark bodies carry net charge ($+4/3e$ homogenously distributed in the diquark and $-1/3e$ in the quark), we choose $r_q = 0.25$ fm and $d = 1.5$ fm, which reproduce charge rms radius of proton (≈ 0.9 fm). In this work, we always choose $\kappa = 0.5$.

For comparison, we also consider a model that we label by the symbol Y in which proton has a highly triangular shape (Fig. 2). The parton density in this case is assumed in the form

$$\rho_Y(\mathbf{r}) = N_g \frac{1-\kappa}{3} \sum_{k=1}^3 \rho_q(\mathbf{r} - \mathbf{r}_k; r_q) + N_g \frac{\kappa}{3} \sum_{k=1}^3 \rho_g \left(\mathcal{R}^{-1} \left[k \frac{2\pi}{3} \right] (\mathbf{r} - \frac{\mathbf{r}_k}{2}); r_q, \frac{r_k}{2} \right), \quad (10)$$

where $\mathbf{r}_1 = (0, \frac{\sqrt{3}}{4}d, -\frac{d}{4})$, $\mathbf{r}_2 = (0, -\frac{\sqrt{3}}{4}d, -\frac{d}{4})$, $\mathbf{r}_3 = (0, 0, \frac{d}{2})$ and $\mathcal{R}[\vartheta]$ is a rotation matrix in yz plane. In this model, the effective quarks are located in the vertices of an equilateral triangle and the gluon tubes connect them with the center of mass of the system. Again we choose $r_q = 0.25$ fm and $d = 1.5$ fm.

The densities of types I and Y are fixed proton structure models in a sense that the only parameters that fluctuate event-by-event are the angles describing proton orientation relative to the direction of its velocity (not explicitly introduced above).

3.2 Gaussian-fluctuating configuration model

We also consider a model with a fluctuating proton configuration, labelled G-f, in which the relative positions of effective quarks differ event-by-event. By generalizing the previous formulas for proton densities, we introduce Gaussian-fluctuating proton density

$$\rho_{G-f}(\mathbf{r}; \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = N_g \frac{1-\kappa}{3} \sum_{k=1}^3 \rho_q(\mathbf{r} - \mathbf{r}_k; r_q) + N_g \frac{\kappa}{3} \sum_{k=1}^3 \rho_g \left[\mathcal{R}^{-1}[\theta_k, \phi_k] (\mathbf{r} - \frac{\mathbf{r}_k}{2}); r_q, \frac{r_k}{2} \right]. \quad (11)$$

In this expression, $\mathcal{R}[\theta, \phi]$ transforms vector $(0, 0, 1)$ into $(\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta)$ and $\mathbf{r}_k = r_k(\cos \phi_k \sin \theta_k, \sin \phi_k \sin \theta_k, \cos \theta_k)$ is the position vector of k -th effective quark.

Following the harmonic oscillator phenomenology, we assume that the probability density of finding a proton whose quarks are in positions $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3$ is Gaussian with an

additional constraint of in the center-of-mass frame ($\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = 0$, obtained by recentering of the quarks),

$$P(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \frac{1}{[(2\pi)^{3/2} R_P^3]^3} e^{-(r_1^2 + r_2^2 + r_3^2)/2R_P^2}. \quad (12)$$

The free parameters in the G-f model are κ , r_q and R_P . Again we set $r_q = 0.25$ fm, which implies the value $R_P = 0.43$ fm, if one wants to obtain the experimental rms proton charge radius.

4 Results

We present the event distribution of eccentricities ϵ_2 and triangularities ϵ_3 for three classes of pp collision models: II, YY and G-f in which colliding protons are described by I, Y or G-f model, respectively (Fig. 3, 4). We also present the RMS values $\sqrt{\langle \epsilon_2^2 \rangle}$ and $\sqrt{\langle \epsilon_3^2 \rangle}$ in certain multiplicity bins (Fig. 5, 6) and the distribution of event multiplicities N (Fig. 7). The reason for presenting RMS and not mean values of anisotropies is because the former correspond hydrodynamically to the actual anisotropic flow coefficients extracted from two particle correlations (we base this approach on [12]). The minimum bias (averaged over all events) $\sqrt{\langle \epsilon_2^2 \rangle_{\text{MB}}}$ are 0.37, 0.40 and 0.31 for II, YY and G-f model, respectively, while $\sqrt{\langle \epsilon_3^2 \rangle_{\text{MB}}} = 0.14, 0.23, 0.15$, correspondingly (these quantities can be calculated using probability densities from Fig. 3 and Fig. 4).

All the calculations are performed by means of a Monte Carlo algorithm. 300 000 events are generated for each class of collision. The statistical weight of an event is given by Eq. (4) and the mean values are calculated according to Eq. (5). In case of I and Y models, we average only over the impact parameter b and over all the possible space orientations of protons during the collision. In case of G-f model, the additional averaging over the positions of effective quarks takes place. The event multiplicity N is estimated by the use of Eq. (6).

We set N_g in such a way that the experimental pp inelastic cross section of 60 mb is reproduced. This is accomplished for N_g in range 7-10. Similarly, the parameter α from Eq. (6) is found to vary between 5 and 11.

5 Conclusions

Our calculations predict quite high values of eccentricities and triangularities in pp collisions. Assuming the hydrodynamic limit in which the scaling between the initial spatial and finite momentum anisotropy is linear and taking $v_2/\epsilon_2 \sim 0.3$ [15], we expect minimum bias $v_2 \sim 0.11, 0.12, 0.09$ for quark-diquark, triangular and Gaussian-fluctuating model. Bożek extracts from the ridge effect v_2 in range of 0.04 - 0.08 [4] which can potentially

be in agreement with our results if viscous effects reducing the scaling factor are taken into account.

Our models predict that one in principle should be able to distinguish quark-diquark (I), triangular (Y) and Gaussian-fluctuating (G-f) proton configurations by extracting the collective flow coefficients v_2 and v_3 from multi-particle correlations. The actual shapes of the collective flow dependence on multiplicity will differ from Fig. 5 and Fig. 6, though it is reasonable to assume that to a first approximation the scaling of v_n with ϵ_n is linear. In general, the anisotropies in collisions of triangular protons, denoted as YY, are much higher than in those of II or G-fG-f. This was expected because irrespective of the orientation of Y protons the collision area is always anisotropic. We find that the values of anisotropy coefficients in YY collisions increase strongly in high multiplicity bins N . Collisions of quark-diquark protons (II) are characterised by relatively large eccentricities ϵ_2 (though not as large as in YY collisions) and small triangularities ϵ_3 . The distributions of anisotropies and the N dependence of their mean values are the most smooth for the G-fG-f collisions, which results from the existence of additional degrees of freedom.

We observe that the fraction of high-multiplicity events in the fixed configuration models is smaller than in the Gaussian-fluctuating model. Our comparison to the experimental data (Fig. 7) indicates that a more realistic model of pp collision should include not just one type, but all II, YY, G-fG-f (and IY, IG-f, YG-f) types of collision configurations with certain probabilities. However, the G-f model alone can approximately describe the experimental results for multiplicity distributions.

Avsar *et al.* [12] using DIPSY Monte Carlo generator predict values of eccentricity which are similar to our result for triangular proton configurations. The distributions of eccentricities within the 'hot spot' model [6] seem to be centered more towards higher values of ϵ_2 than in our result. The probable reason is that the 'hot spot' model lacks any constraint on the relative positions of quarks. It is hard to make any direct comparison with the other mentioned calculations, however all of them [5–13] predict measurable anisotropies in pp collisions.

6 Summary

In view of the great interest in understanding proton structure, it is pointed out that even simple model ideas concerning distribution of quarks and gluons in protons lead to non-trivial multiplicity dependence of initial anisotropies in pp collisions. We illustrate this statement with results obtained for phenomenological models intuitively inspired by RGPEP in QCD and similar ideas. Expecting new data on high-energy pp collisions from the $\sqrt{s} = 14$ TeV runs of the Large Hadron Collider, we believe that detailed studies of the role of proton structure models in description of such collisions may shed new light on the internal structure of protons.

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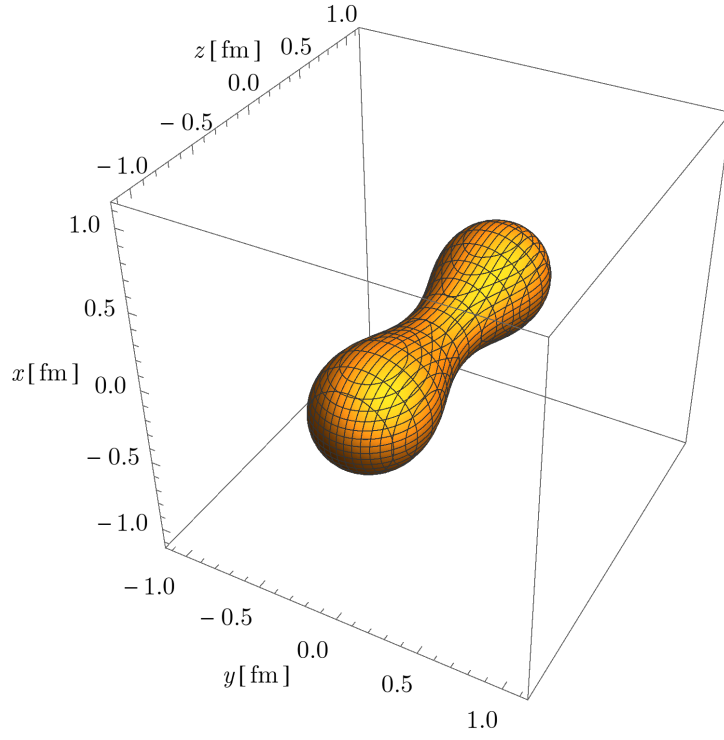


Figure 1: Constant proton density surface for I (quark-diquark) model

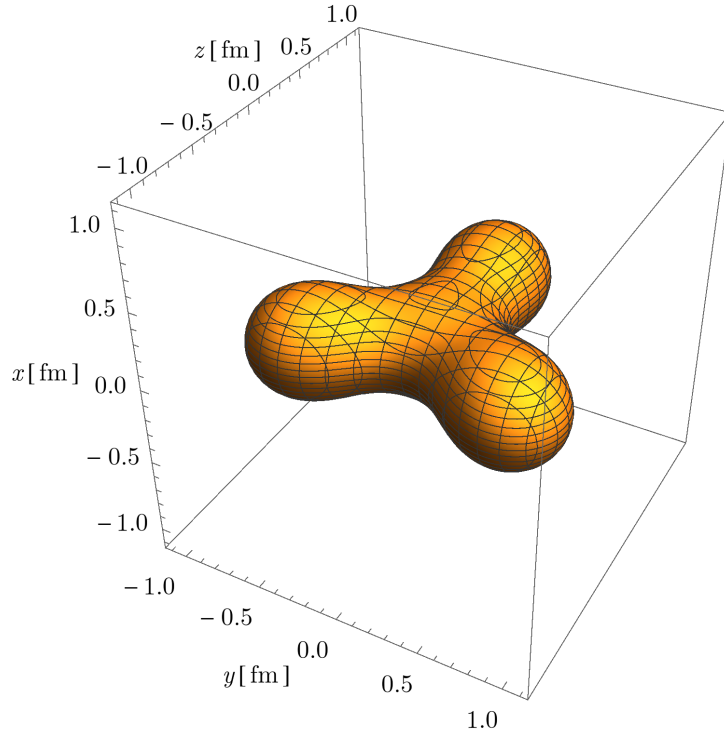


Figure 2: Constant proton density surface for Y model

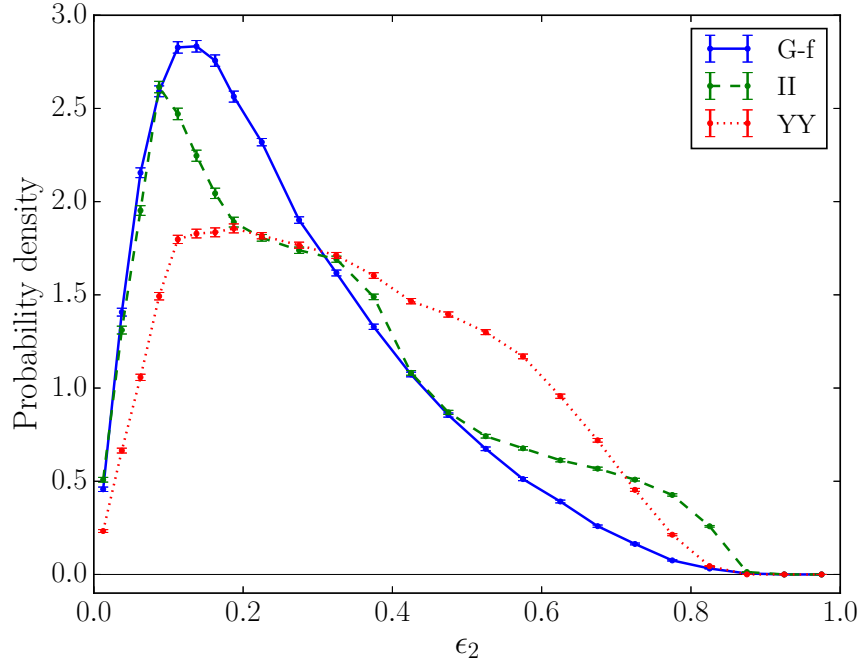


Figure 3: Distribution of event eccentricities for II, YY and Gaussian-fluctuating type (G-f) collisions. The continuous line is reconstructed from the values in particular eccentricity bins and represents the normalized probability density of the occurrence of an event with certain eccentricity. Errors (square roots of variances) follow from basic statistics assuming independent events (applies to all other figures).

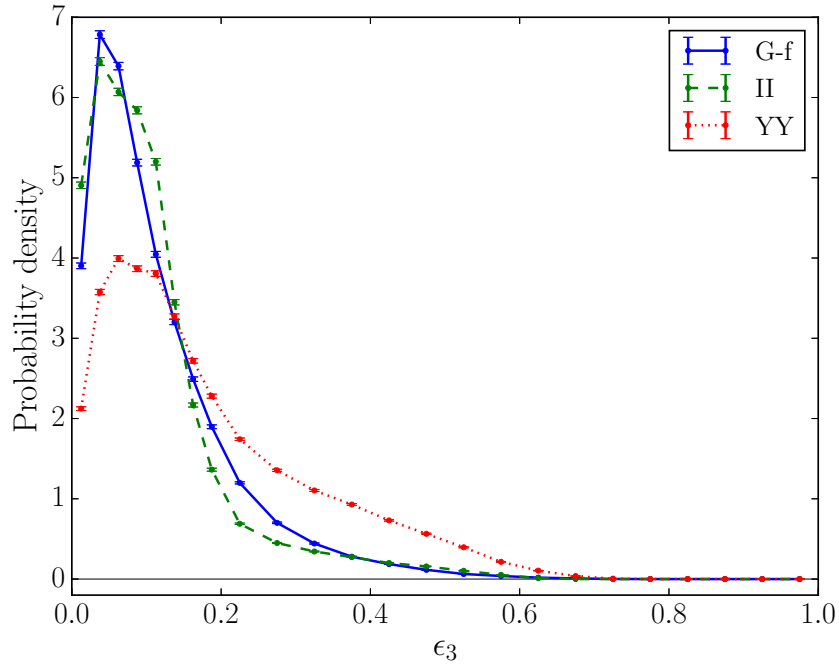


Figure 4: Distribution of event triangularities for II, YY and Gaussian-fluctuating type (G-f) collisions

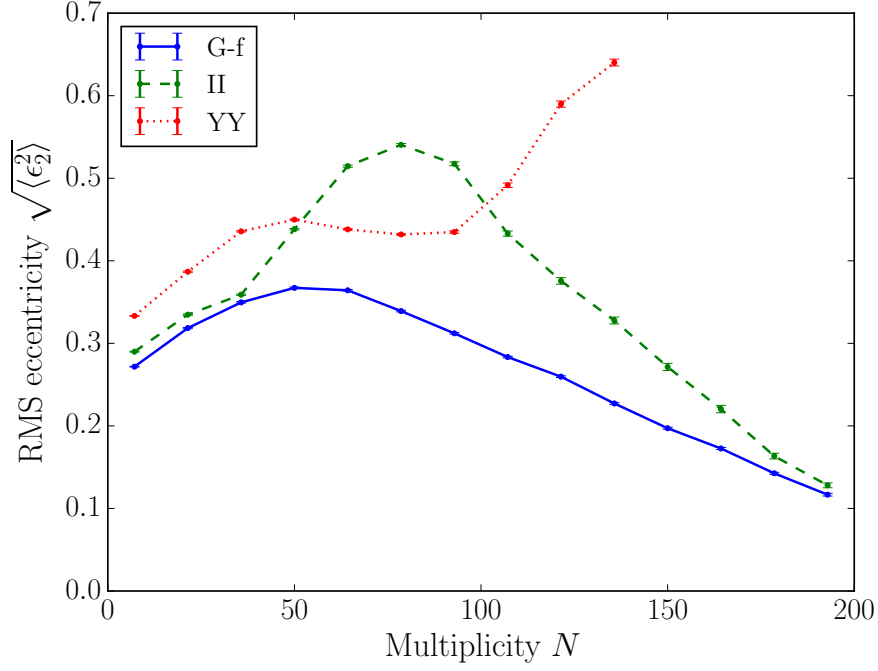


Figure 5: RMS eccentricities in collision multiplicity bins for II, YY and Gaussian-fluctuating type (G-fG-f) collisions

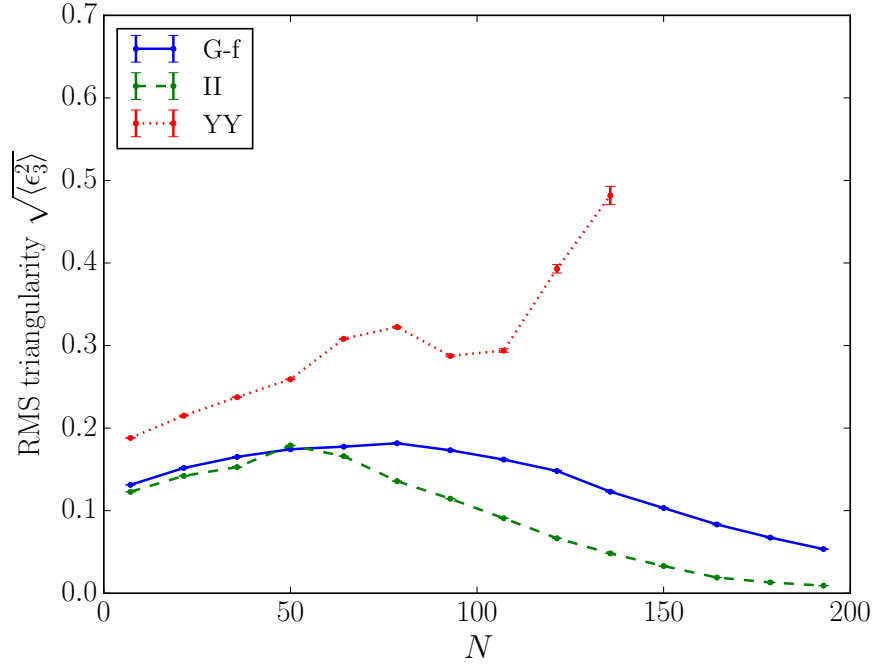


Figure 6: RMS triangularities in collision multiplicity bins for II, YY and Gaussian-fluctuating type (G-fG-f) collisions

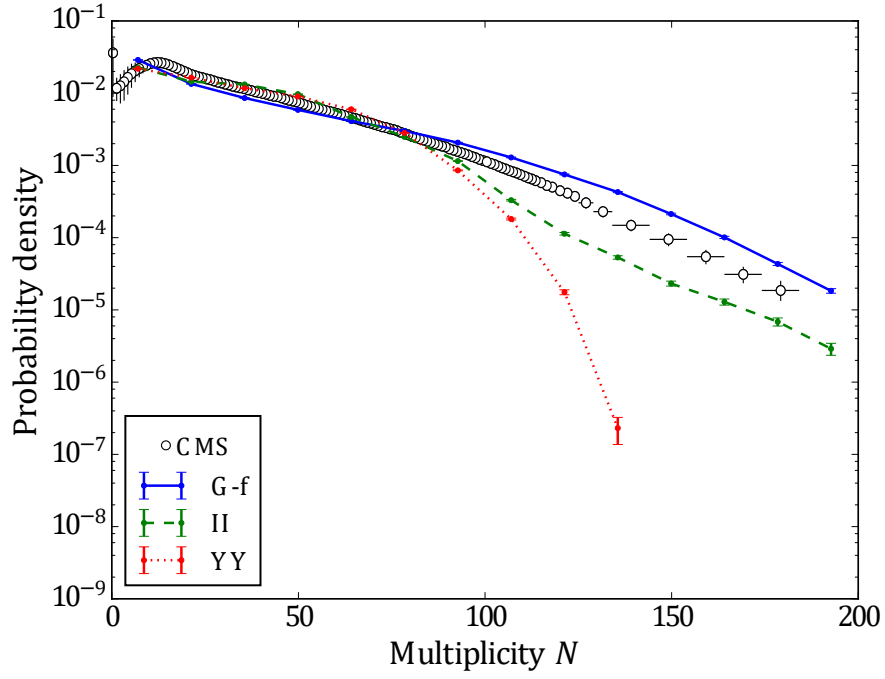


Figure 7: Event multiplicity distributions for II, YY and Gaussian-fluctuating type collisions compared to the experimental charged hadron distribution in $\sqrt{s} = 7 \text{ TeV}$ pp collisions [21] for the pseudorapidity range $|\eta| < 2.4$